

# Praktisk kombinatorik

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## Kort CV

- ▶ Natur, Brännkyrka gymnasium
- ▶ Y-linjen, Linköping
- ▶ Roxen (Web-servrar), EHAND (handdatorer)
- ▶ Doktorsstudier reglerteknik
- ▶ Conemtech, Southpole (linux), Google (WebRTC, Meet)
- ▶ Glasklarteknik AB

Basic, 6502 assembly, Pascal, C, Common LISP, M68k assembly, Objective C, Python, Pike, Scheme, Emacs lisp, C++, tcl, awk, x86 and x86\_64 assembly, CPU microcode, ARM assembly, Javascript, Java, OpenGL Shading Language, Verilog, go.

VIC 20, C64, Amiga, Xerox lisp, Sparc, PC, TKey.

# Tema

Ett program som givet en lista av  $n$  saker och ett tal  $k$ ,  $0 \leq k \leq n$ , skriver ut alla kombinationer av  $k$  saker.

```
$ ./all-k-of-n 2 foo bar baz quux  
foo bar  
foo baz  
bar baz  
foo quux  
bar quux  
baz quux
```

Idéer?

## Sidospår 1: Binomialkoefficienter

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}$$

Sätt att välja  $k$  av  $n$

$$\binom{n}{0} = \binom{n}{n} = 1$$

$$\binom{n}{k} = \underbrace{\binom{n-1}{k-1}}_{\text{med första}} + \underbrace{\binom{n-1}{k}}_{\text{utan första}}$$

$$n > k > 0$$

## Sidospår 1: Binomialkoefficienter

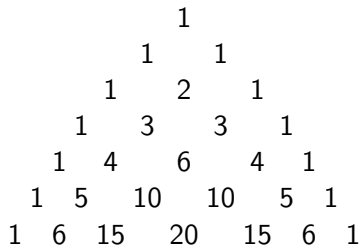
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$$n > k > 0$$



Pascals triangel

## Sidospår 2: Rekursion

En funktion som anropar sig själv.

```
#!/bin/python3
import sys
def fac(n):
    if n == 0:
        return 1
    return n * fac(n-1)

print(fac(int(sys.argv[1])))
```

```
$ ./fac.py 23
```

```
25852016738884976640000
```

# Python 1

```
#!/bin/python3
import itertools, sys

for list in itertools.combinations(sys.argv[2:], int(sys.argv[1])):
    print(' '.join(list))
```

## Python 2

```
#!/bin/python3
import sys

def all_k_of_n(k, args):
    if k == 0 or len(args) == 0:
        return []
    if k == 1:
        return [[x] for x in args]
    first, rest = args[0], args[1:]
    return ([[first] + comb for comb in all_k_of_n(k-1, rest)]
            + all_k_of_n(k, rest))

for list in all_k_of_n(int(sys.argv[1]), sys.argv[2:]):
    print(' '.join(list))
```



## Shell script

```
#!/bin/bash
self="$0"

[[ "$#" -ge 1 ]] || { echo "missing argument" >&2; exit 1; }
k="$1"; shift

[[ "$k" -gt 0 && "$k" -le "$#" ]] || exit 0

if [[ "$k" -eq 1 ]] ; then
    printf "%s\n" "$@"
else
    first="$1"; shift
    "${self}" "$((k - 1))" "$@" | while read -r line ; do
        printf "%s %s\n" "${first}" "${line}" ; done
    "${self}" "$k" "$@"
fi
```

## Scheme

```
#!/usr/bin/guile \  
-e main -s  
!  
(define (main args)  
  (for-each (lambda (list) (display (string-join list)) (newline))  
            (all-k-of-n (string->number (cadr args)) (cddr args))))  
(define (all-k-of-n k args)  
  (cond  
    ((= k 0) '())  
    ((= k 1) (map list args))  
    ((null? args) '())  
    (else (let ((first (car args)) (rest (cdr args)))  
            (append (map (lambda (comb) (cons first comb))  
                        (all-k-of-n (- k 1) rest))  
                    (all-k-of-n k rest))))))
```

## Sidospår 3: Två-komplement

För `uint8_t` har bittarna vikt 1, 2, 4, 8, 16, 32, 64, 128.

För negativa ges översta bitten istället vikt -128.

$$100 = 64 + 32 + 4 = 0b01100100$$

$$\sim 100 = 255 - 100 = 0b10011011$$

flippa *alla* bittar

$$-100 = -128 + 16 + 8 + 4 = 0b10011100$$

flippa bittar över den lägsta 1:an

Två-komplements aritmetik *definierar* negation som  $-x = \sim x + 1$ .

## HAKMEM 175 (Bill Gosper, MIT AI Laboratory 1972)

```
#include <stdint.h>

/* Return next larger number, with the same number of one bits. See
 * https://iamkate.com/code/hakmem-item-175/ */
static uint64_t
next (uint64_t x) {
    uint64_t low, left, changed, right; // x = xxx0 1111 0000
    low = x & -x;                        //      0000 0001 0000
    left = x + low;                      //      xxx1 0000 0000
    changed = x ^ left;                  //      0001 1111 0000
    right = (changed / low) >> 2;        //      0000 0000 0111
    return left | right;                 //      xxx1 0000 0111
}
```

```
#include <stdio.h>
#include <stdlib.h>
#include "next.c"
int main (int argc, char **argv) {
    if (argc < 2 || argc > 65) return 1;
    int k = atoi (argv[1]);
    if (k <= 0) return 0;
    argc -= 2; argv += 2;
    if (k > argc) return 0;

    for (uint64_t mask = ((uint64_t) 1 << k) - 1;
        !(mask >> argc); mask = next (mask)) {
        for (unsigned i = 0; i < argc; i++)
            if ((mask >> i) & 1) printf("%s ", argv[i]);
        printf ("\n");
    }
}
```